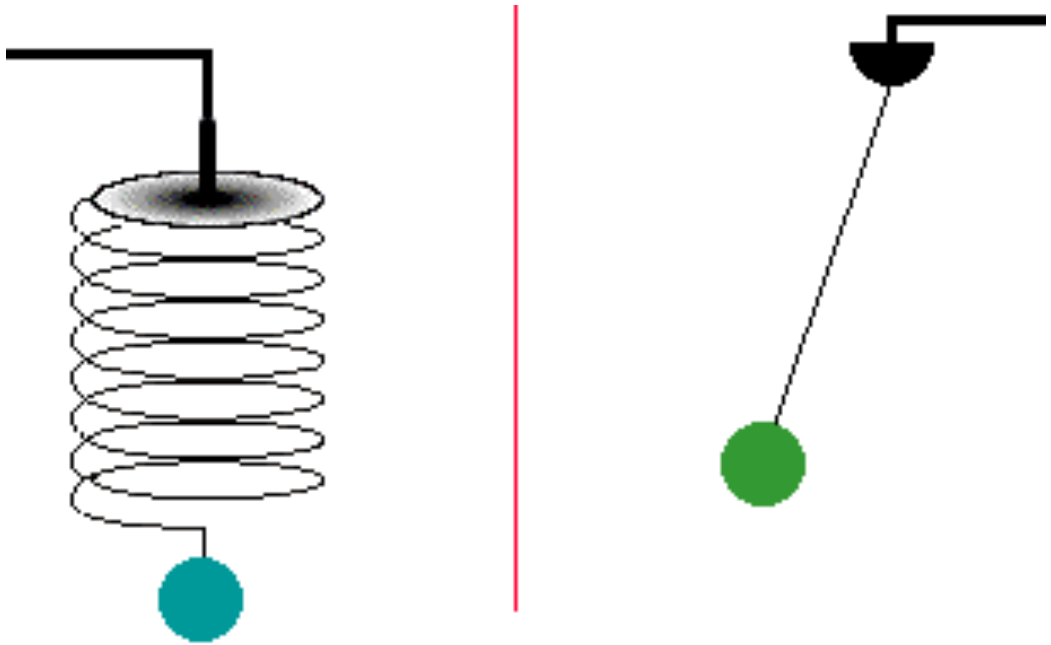
SIMPLE HARMONIC MOTION

SHM **is the motion that occurs when the restoring force acting on an object is proportional to the object's displacement from its equilibrium or rest position.**

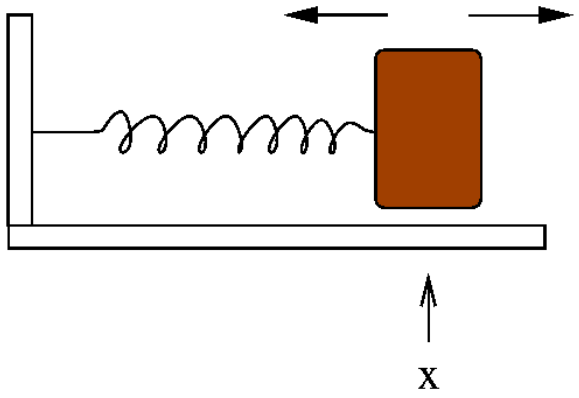


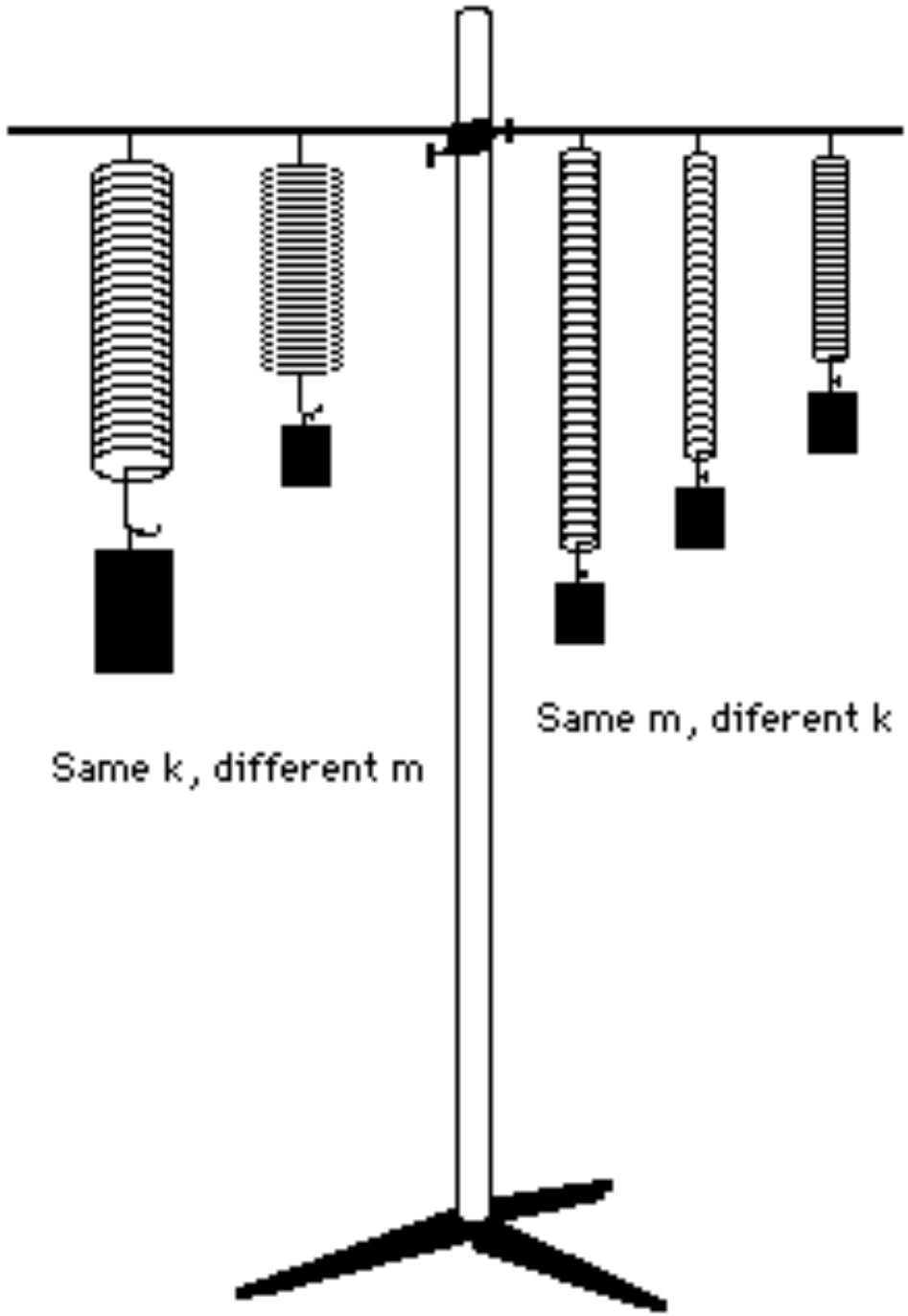
The *period T* of a mass-spring system undergoing simple harmonic motion is the time needed for one complete cycle.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Units: s

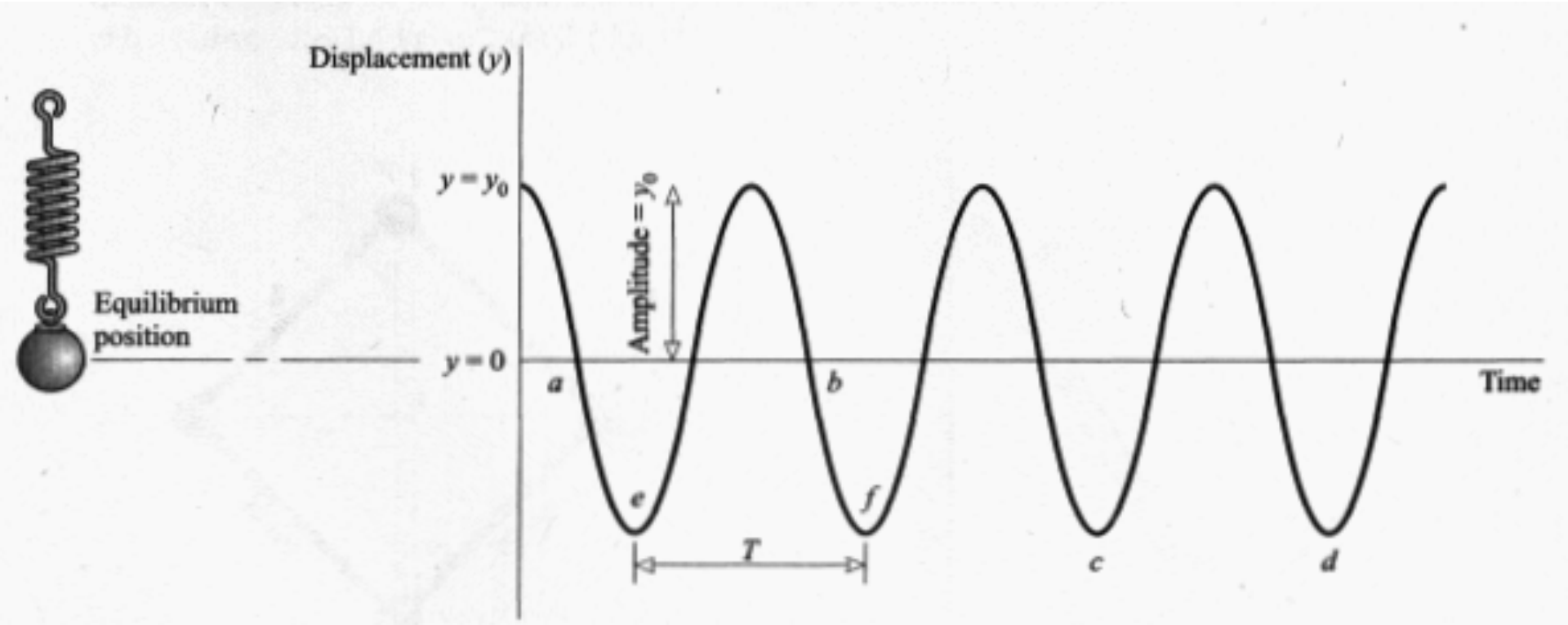
where *k* is the spring's constant




Same k, different m
Same m, diferent k

GRAPH OF SHM

The graph shown below depicts the up and down oscillation of the mass at the end of a spring.
One complete cycle is from a to b, or from c to d.
The time taken for one cycle is *T*, the period.

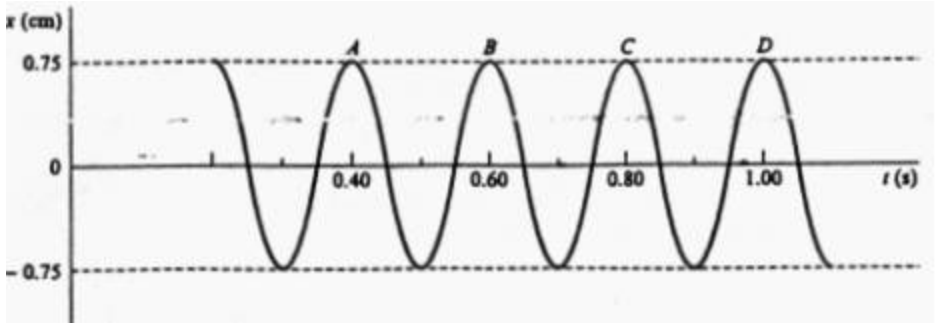


7.1 For the motion shown in the figure, find:

a. Amplitude

b. Period

c. Frequency



a. Amplitude: maximum displacement from equilibrium

A = +- 0.75 cm

b. T = time for one complete cycle

T = 0.2 s

c. $f = 1/T = 1/0.2 =$ 5 Hz

b. What is the period of the oscillation?

m **= 0.1 kg,**
k **= 50 N/m**

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.1}{50}}$$ **= 0.28 s**

c. What is the frequency?

$$f = \frac{1}{T} = \frac{1}{0.28}$$ **= 3.57 Hz**

7.2 A pearl is placed on a spring scale whose spring constant is 362 N/m. If the scale's platform oscillates with a frequency of 1.20 Hz, what is the mass of the pearl?

***k* = 362 N/m**
***f* = 1.2 Hz**

T* = 1/*f
= 1/1.2
= 0.83 s

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T^2 = \frac{4\pi^2 m}{k} \qquad T^2 k = 4\pi^2 m$$

$$m = \frac{T^2 k}{4\pi^2} = \frac{(0.83)^2(362)}{4\pi^2} = \textbf{6.3 kg}$$

THE SIMPLE PENDULUM

A simple pendulum has its entire mass concentrated at the end of a string. It undergoes SHM provided that the arc through which it travels is only a few degrees. The *period* of a simple pendulum of length *L* is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Units: s

where *g* is the acceleration due to gravity

7.3 Find the frequency of a simple pendulum 20 cm long.

***L* = 0.2 m**

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.2}{10}} = 0.89 \text{ s}$$

$$f = \frac{1}{T} = \frac{1}{0.89} = 1.1 \text{ Hz}$$

7.4 A pendulum 1 m long oscillates 30 times per minute in a certain location. What is the value of *g* there?

***L* = 1 m**
***f* = 30 rpm**

$$\left(\frac{30\ \text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\,\text{sec}}\right) = \textbf{0.5 Hz}$$

$$T = \frac{1}{f} = \frac{1}{0.5} = \textbf{2 s}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1)}{2^2} = \textbf{9.86 m/s}^2$$