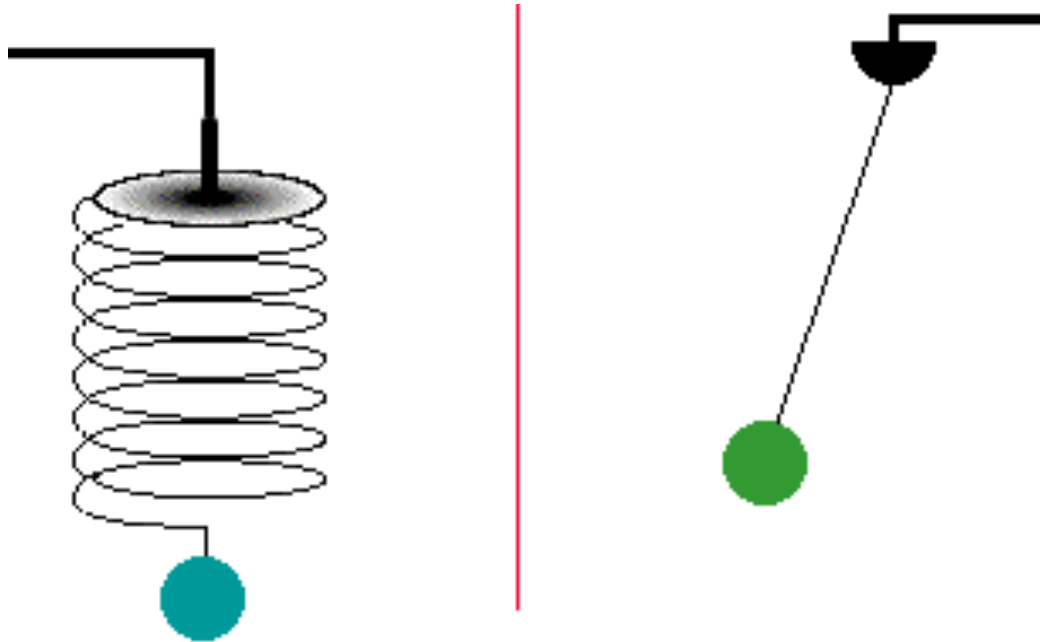


SIMPLE HARMONIC MOTION

SHM is the motion that occurs when the restoring force acting on an object is proportional to the object's displacement from its equilibrium or rest position.

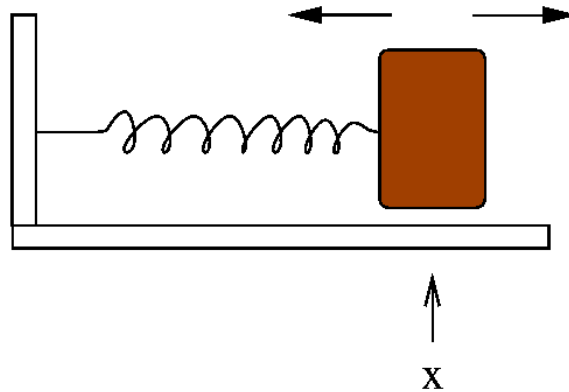


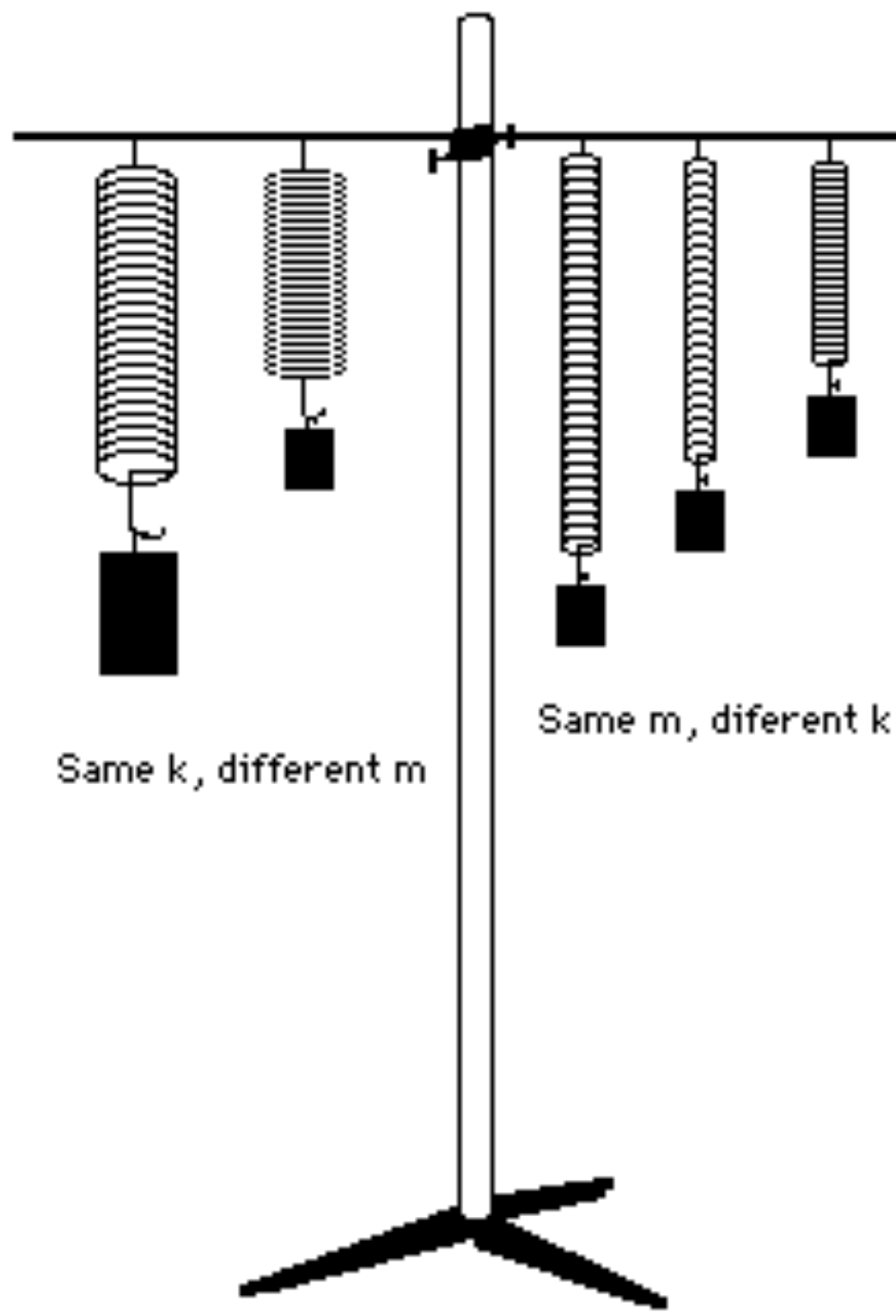
The **period** T of a mass-spring system undergoing simple harmonic motion is the time needed for one complete cycle.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Units: s

where k is the spring's constant





Same k , different m

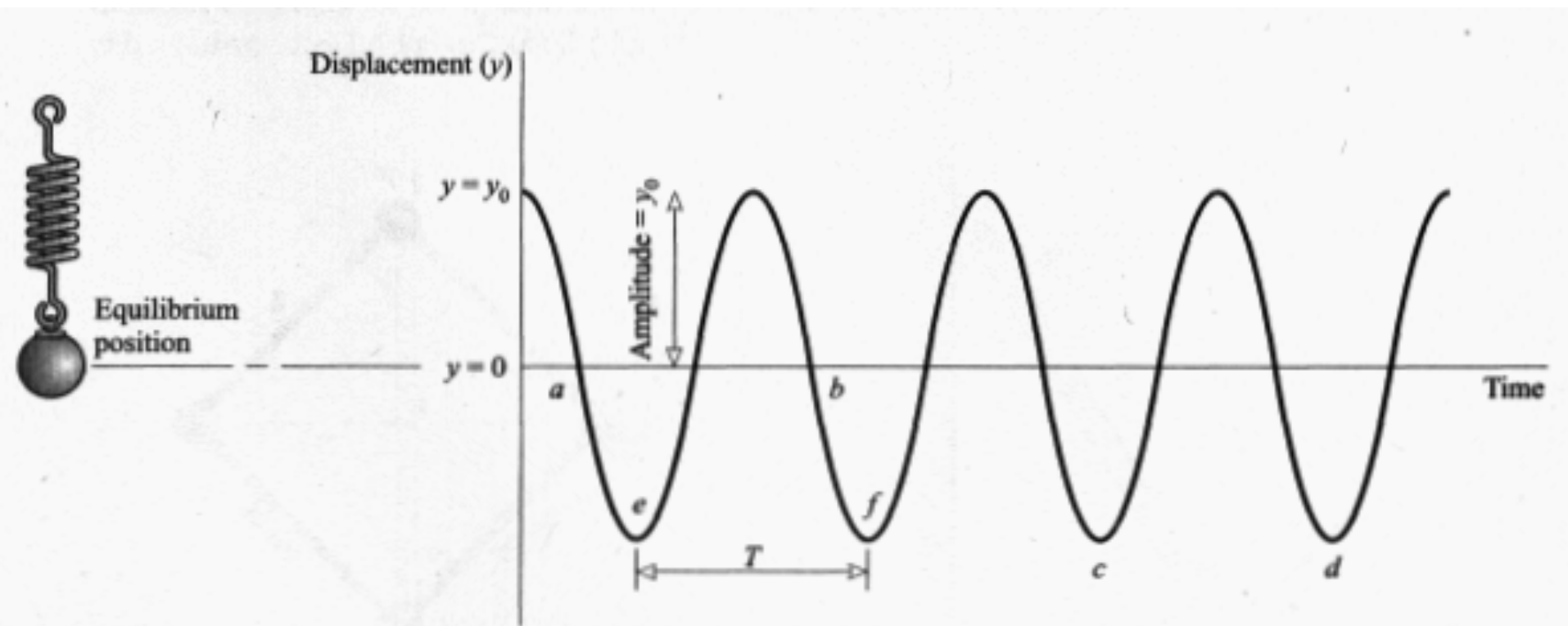
Same m , different k

GRAPH OF SHM

The graph shown below depicts the up and down oscillation of the mass at the end of a spring.

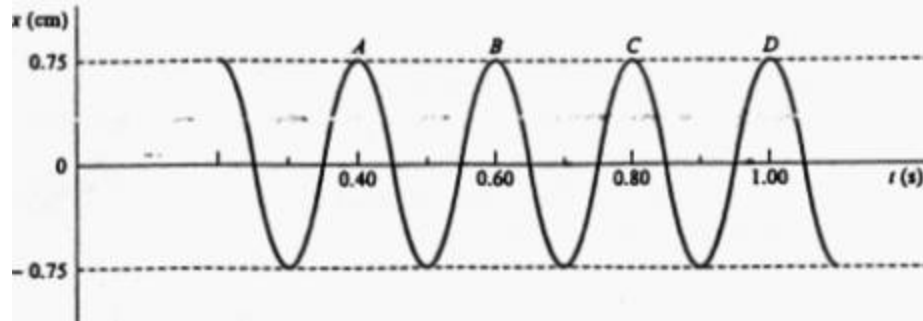
One complete cycle is from **a** to **b**, or from **c** to **d**.

The time taken for **one cycle** is T , the period.



7.1 For the motion shown in the figure, find:

- a.** Amplitude
- b.** Period
- c.** Frequency



a. Amplitude: maximum displacement from equilibrium

$$A = \pm 0.75 \text{ cm}$$

b. T = time for one complete cycle

$$T = 0.2 \text{ s}$$

c. $f = 1/T = 1/0.2 = 5 \text{ Hz}$

b. What is the period of the oscillation?

$$m = 0.1 \text{ kg,}$$
$$k = 50 \text{ N/m}$$

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.1}{50}} = 0.28 \text{ s}$$

c. What is the frequency?

$$f = \frac{1}{T} = \frac{1}{0.28} = 3.57 \text{ Hz}$$

7.2 A pearl is placed on a spring scale whose spring constant is 362 N/m. If the scale's platform oscillates with a frequency of 1.20 Hz, what is the mass of the pearl?

$$k = 362 \text{ N/m}$$

$$f = 1.2 \text{ Hz}$$

$$\begin{aligned} T &= 1/f \\ &= 1/1.2 \\ &= 0.83 \text{ s} \end{aligned}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$T^2 = \frac{4\pi^2 m}{k} \quad T^2 k = 4\pi^2 m$$

$$m = \frac{T^2 k}{4\pi^2} = \frac{(0.83)^2 (362)}{4\pi^2} = 6.3 \text{ kg}$$

THE SIMPLE PENDULUM

A simple pendulum has its entire mass concentrated at the end of a string. It undergoes **SHM** provided that the arc through which it travels is only a few degrees. The *period* of a simple pendulum of length L is given by:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Units: s

where g is the acceleration due to gravity



7.3 Find the frequency of a simple pendulum 20 cm long.

$$L = 0.2 \text{ m}$$

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{0.2}{10}} = 0.89 \text{ s}$$

$$f = \frac{1}{T} = \frac{1}{0.89} = 1.1 \text{ Hz}$$

7.4 A pendulum 1 m long oscillates 30 times per minute in a certain location. What is the value of g there?

$$\begin{array}{l} L = 1 \text{ m} \\ f = 30 \text{ rpm} \end{array} \quad \left(\frac{30 \text{ rev}}{\text{min}} \right) \left(\frac{1 \text{ min}}{60 \text{ sec}} \right) = \mathbf{0.5 \text{ Hz}}$$

$$T = \frac{1}{f} = \frac{1}{0.5} = \mathbf{2 \text{ s}} \quad T = 2\pi \sqrt{\frac{L}{g}}$$

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2 (1)}{2^2} = \mathbf{9.86 \text{ m/s}^2}$$